7.4 Half-Angle Identities

LT44: I can use the double angle and half-angle identities to find exact values and prove other identities.

*NOTE: When applying a half-angle identity, the quadrant that the terminal side of the angle lies in must first be determined so that the correct sign before the radical can be chosen.

Ex $\cos\frac{\pi}{8}$ x2

$\cos\left(\frac{\frac{\pi}{4}}{2}\right) = \sqrt{\frac{1+\cos\frac{\pi}{4}}{2}}$

$\frac{\pi}{8}$ is in Quad. I

$\therefore (+)$

$= \sqrt{\frac{(1+\frac{\sqrt{2}}{2})2}{(2)2}}$

$= \sqrt{\frac{2+\sqrt{2}}{4}}$

$= \frac{\sqrt{2+\sqrt{2}}}{2}$

Ex $\tan 15^\circ$ x2

$15^\circ \rightarrow$ Quad I

$\therefore (+)$

$\tan\left(\frac{30}{2}\right) = \sqrt{\frac{1-\cos 30}{1+\cos 30}}$

$= \sqrt{\frac{(1-\frac{\sqrt{3}}{2})2}{(1+\frac{\sqrt{3}}{2})2}}$

$= \sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}}}$

$= \sqrt{\frac{(2-\sqrt{3})^2}{4-3}} = \sqrt{\frac{(2-\sqrt{3})^2}{1}}$

$= \frac{\sqrt{(2-\sqrt{3})^2}}{1}$

$= 2-\sqrt{3}$